



Generalized tri-bimaximal neutrino mixing and its sensitivity to radiative corrections

Shu Luo, Zhi-Zhong Xing^{*}

CCAST (World Laboratory), PO Box 8730, Beijing 100080, China

Institute of High Energy Physics, Chinese Academy of Sciences, PO Box 918, Beijing 100049, China¹

Received 8 September 2005; received in revised form 25 October 2005; accepted 26 October 2005

Available online 9 November 2005

Editor: T. Yanagida

Abstract

We argue that the tri-bimaximal neutrino mixing pattern V_0 or its generalized form V'_0 , which includes two arbitrary Majorana phases of CP violation, may result from an underlying flavor symmetry at a superhigh energy scale close to the seesaw scale ($\sim 10^{14}$ GeV). Taking the working assumption that three neutrino masses are nearly degenerate, we calculate radiative corrections to V_0 and V'_0 in their evolution down to the electroweak scale ($\sim 10^2$ GeV). Three mixing angles of V_0 or V'_0 are essentially stable against radiative corrections in the standard model (SM). In the minimal supersymmetric standard model (MSSM), however, V_0 is in general disfavored and V'_0 can be compatible with current neutrino oscillation data if its two Majorana phases α_1 and α_2 are properly fine-tuned. We also find that it is possible to radiatively generate the CP-violating phase δ from α_1 and α_2 , and δ may keep on staying at its quasi-fixed point in either the SM or the MSSM.

© 2005 Elsevier B.V. Open access under CC BY license.

PACS: 14.60.Pq; 13.10.+q; 25.30.Pt

1. Current solar [1], atmospheric [2], reactor [3] and accelerator [4] neutrino experiments have provided us with very convincing evidence for the existence of neutrino oscillations, a quantum phenomenon which can naturally occur if neutrinos are massive and lepton flavors are mixed. The property of lepton flavor mixing can be described by a 3×3 unitary matrix V . A parametrization of V , advocated by the Particle Data Group [5], reads as

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (1)$$

where $c_{ij} \equiv \cos \theta_{ij}$ and $s_{ij} \equiv \sin \theta_{ij}$ (for $ij = 12, 23$ and 13). The phase parameters α_1 and α_2 are usually referred as to the Majorana CP-violating phases, because they have nothing to do with CP or T violation in the neutrino–neutrino and antineutrino–antineutrino oscillations. A global analysis of the present experimental data yields [6] $30^\circ \leq \theta_{12} \leq 38^\circ$, $36^\circ \leq \theta_{23} \leq 54^\circ$ and $0^\circ \leq \theta_{13} \leq 10^\circ$ as well as $\Delta m_{21}^2 \equiv m_2^2 - m_1^2 = (7.2\text{--}8.9) \times 10^{-5} \text{ eV}^2$ and $\Delta m_{32}^2 \equiv m_3^2 - m_2^2 = \pm(1.7\text{--}3.3) \times 10^{-3} \text{ eV}^2$ at the 99% confidence level. In contrast, three phases of V are entirely unrestricted. A variety of new neutrino experiments are underway, not only to detect the smallest flavor mixing angle θ_{13} and the phase parameter δ , but also to constrain the Majorana phases α_1 and α_2 .

To interpret the observed neutrino mass spectrum and the observed bilarge neutrino mixing pattern, many theoretical and phenomenological models have been proposed and discussed [7]. A category of models or ansätze have attracted some particular

^{*} Corresponding author.

E-mail address: xingzz@ihep.ac.cn (Z.-Z. Xing).

¹ Mailing address.

attention, because they can give rise to the so-called tri-bimaximal neutrino mixing pattern [8]:

$$V_0 = \begin{pmatrix} \sqrt{6}/3 & \sqrt{3}/3 & 0 \\ -\sqrt{6}/6 & \sqrt{3}/3 & \sqrt{2}/2 \\ \sqrt{6}/6 & -\sqrt{3}/3 & \sqrt{2}/2 \end{pmatrix}. \quad (2)$$

Comparing between Eqs. (1) and (2), one may immediately observe that V_0 has $\theta_{12} \approx 35.3^\circ$, $\theta_{23} = 45^\circ$, $\theta_{13} = 0^\circ$ and $\alpha_1 = \alpha_2 = 0^\circ$. The phase parameter δ is not well defined in V_0 , as a consequence of $\theta_{13} = 0^\circ$. The results $\sin^2 2\theta_{12} = 8/9$ and $\sin^2 2\theta_{23} = 1$ are in good agreement with current data on solar and atmospheric neutrino oscillations. It is straightforward to generalize V_0 in order to include two arbitrary Majorana phases,

$$V'_0 = \begin{pmatrix} \sqrt{6}/3 & \sqrt{3}/3 & 0 \\ -\sqrt{6}/6 & \sqrt{3}/3 & \sqrt{2}/2 \\ \sqrt{6}/6 & -\sqrt{3}/3 & \sqrt{2}/2 \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (3)$$

Although V_0 and V'_0 have the same impact on neutrino oscillations, their consequences on the neutrinoless double-beta decay are certainly different. In this sense, we refer to V'_0 as the *generalized* tri-bimaximal neutrino mixing pattern.

Such a special neutrino mixing pattern is in general expected to result from an underlying flavor symmetry (e.g., the discrete non-Abelian symmetry A_4 [9–12]) and its spontaneous or explicit breaking. The latter is always necessary, because a flavor symmetry itself cannot reproduce the observed lepton mass spectra and predict the realistic lepton mixing pattern simultaneously [13]. Specific and compelling constructions of this kind of flavor symmetry breaking are a real challenge and have been lacking, although some attempts have been made in the literature [7]. The energy scale, at which a proper flavor symmetry can be realized, may be considerably higher than the electroweak scale ($\Lambda_{EW} \sim 10^2$ GeV). This new physics (NP) scale Λ_{NP} has actually been identified with other known scales in some model-building works [7], including the grand-unification-theory scale ($\Lambda_{GUT} \sim 10^{16}$ GeV) or the seesaw scale ($\Lambda_{SS} \sim 10^{14}$ GeV). In this case, radiative corrections to the relevant model parameters between Λ_{EW} and Λ_{NP} must be taken into account [14].

One may then ask whether the generalized tri-bimaximal neutrino mixing pattern is stable or not against radiative corrections, if it is derived from an underlying (broken) flavor symmetry within an unspecified mechanism at Λ_{NP} ($\gg \Lambda_{EW}$). The main purpose of this Letter is just to answer this question by considering both the standard model (SM) and its minimal supersymmetric extension (MSSM) below Λ_{NP} . The only effective dimension-5 operator of light Majorana neutrinos reads as

$$\mathcal{L}_\nu = \frac{1}{2} \kappa_{ij} (H \cdot L_i) (H \cdot L_j) + \text{h.c.}, \quad (4)$$

where H denotes the SM Higgs (or the MSSM Higgs with the appropriate hypercharge), L_i (for $i = 1, 2, 3$) stand for the leptonic $SU(2)_L$ doublets, and κ is a symmetric neutrino coupling matrix. After spontaneous gauge symmetry breaking at Λ_{EW} , we arrive at the effective neutrino mass matrix $M_\nu = v^2 \kappa$ (SM) or $M_\nu = v^2 \kappa \sin^2 \beta$ (MSSM), where $v \approx 174$ GeV and $\tan \beta$ is the ratio of the vacuum expectation values of two Higgs fields in the MSSM. Between Λ_{EW} and Λ_{NP} , the most important radiative correction to κ is proportional to $\ln(\Lambda_{NP}/\Lambda_{EW})$ and can be evaluated by using the one-loop renormalization group equations (RGEs) [14]. It is then possible to calculate the RGE effects on the lepton flavor mixing parameters analytically and numerically.

In the working assumption that three neutrino masses are nearly degenerate, we are going to calculate radiative corrections to V_0 and V'_0 . We show that both V_0 and V'_0 are stable against radiative corrections in the SM, but only V'_0 with the proper fine-tuning of $(\alpha_2 - \alpha_1)$ is allowed in the MSSM. In addition, the CP-violating parameter δ can be radiatively generated from α_1 and α_2 . A peculiar feature of δ is that it may keep on staying at its quasi-fixed point in both the SM and the MSSM.

2. Taking account of the seesaw mechanism [15] as a natural idea to understand the origin of neutrino masses and lepton flavor mixing, we assume the new physics (i.e., new flavor symmetry) scale Λ_{NP} is close to the seesaw scale $\Lambda_{SS} \sim 10^{14}$ GeV. Below Λ_{NP} , the effective neutrino coupling matrix κ obeys the one-loop RGE [16]²:

$$16\pi^2 \frac{d\kappa}{dt} = \alpha \kappa + C [(Y_l Y_l^\dagger) \kappa + \kappa (Y_l Y_l^\dagger)^T], \quad (5)$$

in which $t \equiv \ln(\mu/\Lambda_{NP})$ with μ being an arbitrary renormalization scale below Λ_{NP} but above Λ_{EW} . In the SM, $C = -1.5$ and $\alpha \approx -3g_2^2 + 6y_t^2 + \lambda$; and in the MSSM, $C = 1$ and $\alpha \approx -1.2g_1^2 - 6g_2^2 + 6y_t^2$, where g_1 and g_2 denote the gauge couplings, y_t stands for the top-quark Yukawa coupling, and λ represents the Higgs self-coupling in the SM [16]. In the flavor basis where the charged-lepton Yukawa coupling matrix is diagonal and real (positive), we have $\kappa = V \bar{\kappa} V^T$ with $\bar{\kappa} = \text{Diag}\{\kappa_1, \kappa_2, \kappa_3\}$. The

² Note that $\Lambda_{NP} \sim \Lambda_{SS}$ is an effective working assumption, in which the possible mass hierarchy of three heavy right-handed neutrinos N_i (for $i = 1, 2, 3$) is omitted. If $\Lambda_{NP} \sim \Lambda_{GUT}$ ($> \Lambda_{SS}$) is assumed and the mass hierarchy of N_i is considered, then very strong seesaw threshold effects may appear in the RGE evolution of relevant model parameters (see Ref. [17] for detailed discussions).

neutrino masses at Λ_{EW} are given by $m_i = v^2 \kappa_i$ (SM) or $m_i = v^2 \kappa_i \sin^2 \beta$ (MSSM). One can then derive the RGEs for $(\kappa_1, \kappa_2, \kappa_3)$, $(\theta_{12}, \theta_{23}, \theta_{13})$ and $(\delta, \alpha_1, \alpha_2)$ from Eq. (5), just like the previous works done in Refs. [16–19].

To be specific, we assume the masses of three Majorana neutrinos are nearly degenerate; i.e., $m_1 \approx m_2 \approx m_3$. Such a working assumption makes sense at least for two practical reasons: (1) it might hint at the slight breaking of an exact $S(3)$ permutation symmetry [20] or other possible flavor symmetries, from which the tri-bimaximal neutrino mixing pattern can naturally arise; and (2) more significant RGE running effects on three mixing angles and three CP-violating phases can manifest themselves in this interesting case. Furthermore, it is helpful to make some analytical approximations for the results obtained in Refs. [16–19] by taking account of the smallness of $\sin \theta_{13}$ and $\Delta m_{21}^2 / \Delta m_{32}^2$. We arrive at

$$\frac{dm_1}{dt} \approx \frac{m_1}{16\pi^2} (\alpha + 2Cy_\tau^2 s_{12}^2 s_{23}^2), \quad \frac{dm_2}{dt} \approx \frac{m_2}{16\pi^2} (\alpha + 2Cy_\tau^2 c_{12}^2 s_{23}^2), \quad \frac{dm_3}{dt} \approx \frac{m_3}{16\pi^2} (\alpha + 2Cy_\tau^2 c_{23}^2) \quad (6)$$

to a good degree of accuracy, where y_τ denotes the tau-lepton Yukawa coupling. Given the approximate degeneracy of three neutrino masses, the RGEs of $(\theta_{12}, \theta_{23}, \theta_{13})$ and $(\delta, \alpha_1, \alpha_2)$ in Refs. [16–19] are simplified to

$$\begin{aligned} \frac{d\theta_{12}}{dt} &\approx -\frac{Cy_\tau^2}{4\pi^2} \frac{m_1^2}{\Delta m_{21}^2} c_{12} s_{12} s_{23}^2 \cos^2 \frac{\alpha_2 - \alpha_1}{2}, \\ \frac{d\theta_{23}}{dt} &\approx -\frac{Cy_\tau^2}{4\pi^2} \frac{m_1^2}{\Delta m_{32}^2} c_{23} s_{23} \left(c_{12}^2 \cos^2 \frac{\alpha_2}{2} + s_{12}^2 \cos^2 \frac{\alpha_1}{2} \right), \\ \frac{d\theta_{13}}{dt} &\approx -\frac{Cy_\tau^2}{8\pi^2} \frac{m_1^2}{\Delta m_{32}^2} c_{12} s_{12} c_{23} s_{23} [\cos(\delta + \alpha_2) - \cos(\delta + \alpha_1)]; \end{aligned} \quad (7)$$

as well as

$$\begin{aligned} \frac{d\delta}{dt} &\approx \frac{Cy_\tau^2}{8\pi^2} \left[\frac{m_1^2}{\Delta m_{32}^2} \cdot \frac{c_{12} s_{12} c_{23} s_{23}}{s_{13}} [\sin(\delta + \alpha_2) - \sin(\delta + \alpha_1) + \chi] + \frac{m_1^2}{\Delta m_{21}^2} s_{23}^2 \sin(\alpha_2 - \alpha_1) \right], \\ \frac{d\alpha_1}{dt} &\approx -\frac{Cy_\tau^2}{4\pi^2} \frac{m_1^2}{\Delta m_{21}^2} c_{12}^2 s_{23}^2 \sin(\alpha_2 - \alpha_1), \quad \frac{d\alpha_2}{dt} \approx -\frac{Cy_\tau^2}{4\pi^2} \frac{m_1^2}{\Delta m_{21}^2} s_{12}^2 s_{23}^2 \sin(\alpha_2 - \alpha_1), \end{aligned} \quad (8)$$

where

$$\chi = \frac{\Delta m_{21}^2}{\Delta m_{32}^2} [\sin(\delta + \alpha_1) + \sin \delta]. \quad (9)$$

Note that the χ -term is not negligible only in the special case that $\alpha_1 \approx \alpha_2$ holds and s_{13} is extremely small. Some qualitative comments on Eqs. (7) and (8) are in order.

(a) The mixing angle θ_{12} is in general more sensitive to radiative corrections than θ_{23} and θ_{13} [16,18]. Given $\theta_{12} \approx 35.3^\circ$ as a result of the tri-bimaximal neutrino mixing at Λ_{NP} , the RGE running effect has to be sufficiently suppressed such that θ_{12} can finally run into the experimentally-allowed range $30^\circ \leq \theta_{12} \leq 38^\circ$ at low energies. This requirement is certainly satisfied in the SM with $m_1 \sim \mathcal{O}(0.1 \text{ eV})$, in which θ_{12} slightly decreases in the RGE evolution from Λ_{NP} to Λ_{EW} . In the MSSM, however, θ_{12} must evolve to a bigger value at Λ_{EW} . Hence the fine-tuning of $(\alpha_2 - \alpha_1)$ is necessary for large values of $\tan \beta$, so as to keep the evolution effect of θ_{12} insignificant [16,18,21]. One can see that two Majorana phases of V'_0 play a very non-trivial role in the calculation of radiative corrections. In other words, the tri-bimaximal neutrino mixing pattern V_0 and its generalized counterpart V'_0 are distinguishable in model building by taking into account their different RGE running behaviors.

(b) Different from θ_{12} , the mixing angles θ_{23} and θ_{13} are expected to be less sensitive to radiative corrections. Hence θ_{23} at Λ_{EW} may slightly deviate from its initial value $\theta_{23} = 45^\circ$ at Λ_{NP} as a consequence of the RGE running. On the other hand, θ_{13} can be radiatively generated, although its value at Λ_{EW} must be rather small. Note that the tri-bimaximal neutrino mixing pattern V_0 keeps CP-conserving in the RGE evolution from Λ_{NP} to Λ_{EW} . As for the generalized tri-bimaximal neutrino mixing scenario V'_0 , it is possible to generate both the mixing angle θ_{13} and the CP-violating phase δ radiatively.³ The latter results from two non-trivial Majorana phases (or one of them) in the RGE of δ . This observation implies that the RGE-corrected V'_0 may give rise to a non-vanishing Jarlskog invariant [22] at Λ_{EW} , leading to observable CP violation in neutrino oscillations.

(c) It should be noted that δ is not well defined at Λ_{NP} , where θ_{13} is exactly vanishing in either V_0 or V'_0 . This point can clearly be seen from Eq. (8), in which the derivative of δ diverges in the $\theta_{13} \rightarrow 0$ limit. Nevertheless, it has been shown in Ref. [18] that there exists an analytic continuation condition, which assures $d\delta/dt$ to keep finite and the running of δ to be extended continuously even when θ_{13} approaches zero. Hence θ_{13} and δ can simultaneously be generated from the RGE running effects in the generalized

³ For a more generic study of this problem, we refer readers to the works done by Casas et al. in Ref. [16], Antusch et al. in Ref. [18], and Luo et al. in Ref. [19].

tri-bimaximal neutrino mixing scenario. A peculiar RGE behavior of δ is that it can keep on staying at its quasi-fixed point just below Δ_{NP} , as one can see in the subsequent numerical examples.

3. Now let us quantitatively illustrate radiative corrections to V_0 and V'_0 by taking a few numerical examples.⁴ The eigenvalues of Y_l at Δ_{NP} are chosen in such a way that they can correctly run to their low-energy values. We typically take $m_1 \sim 0.2$ eV, which is consistent with the working assumption $m_1 \approx m_2 \approx m_3$ made above. The initial values of three mixing angles at Δ_{NP} are fixed: $\theta_{12} \approx 35.3^\circ$, $\theta_{23} = 45^\circ$ and $\theta_{13} = 0^\circ$, predicted by V_0 or V'_0 . Two unknown Majorana phases in V'_0 are adjustable in our numerical calculations, in which we choose $m_H = 140$ GeV (SM) or $\tan \beta = 10$ (MSSM) as a typical and instructive input. The primary results are shown in Tables 1 and 2 together with Figs. 1 and 2. Some discussions are in order.

(A) *In the SM.* It is demonstrated that the RGE running effects on three mixing angles (θ_{12} , θ_{23} and θ_{13}) are small enough in the SM, thus either V_0 or V'_0 at Δ_{NP} can agree with current neutrino oscillation data at low energies. For V_0 , CP conservation keeps to hold in the RGE evolution from Δ_{NP} to Δ_{EW} (see case I in Table 1). But for V'_0 , the radiative generation of δ can always take place, only if α_1 or α_2 is initially non-vanishing (see cases II and III in Table 1). Because of $\theta_{13} = 0^\circ$ at Δ_{NP} , an extraordinarily large

Table 1
Radiative corrections to V_0 (case I) and V'_0 (cases II and III) from $\Delta_{\text{NP}} \sim 10^{14}$ GeV to $\Delta_{\text{EW}} \sim 10^2$ GeV in the SM. The Higgs mass $m_H = 140$ GeV has typically been input in our numerical calculation. Note that δ is not well defined in the $\theta_{13} = 0$ limit at Δ_{NP} , but its running behavior is independent of this ambiguity and is fixed by the initial values of α_1 and α_2

Parameter	Case I (V_0)		Case II (V'_0)		Case III (V'_0)	
	Δ_{NP}	Δ_{EW}	Δ_{NP}	Δ_{EW}	Δ_{NP}	Δ_{EW}
m_1 (eV)	0.310	0.200	0.310	0.200	0.310	0.200
Δm_{21}^2 (10^{-5} eV ²)	18.83	7.91	18.83	7.91	18.83	7.91
Δm_{31}^2 (10^{-3} eV ²)	5.31	2.21	5.31	2.21	5.31	2.21
θ_{12}	35.26°	34.48°	35.26°	35.24°	35.26°	34.48°
θ_{23}	45.0°	44.94°	45.0°	44.97°	45.0°	44.96°
θ_{13}	0°	0.001°	0°	0.0288°	0°	0.0008°
δ	–	0°	–	90.61°	–	140.0°
α_1	0°	0°	260.0°	260.38°	80.0°	80.0°
α_2	0°	0°	100.0°	100.19°	80.0°	80.0°

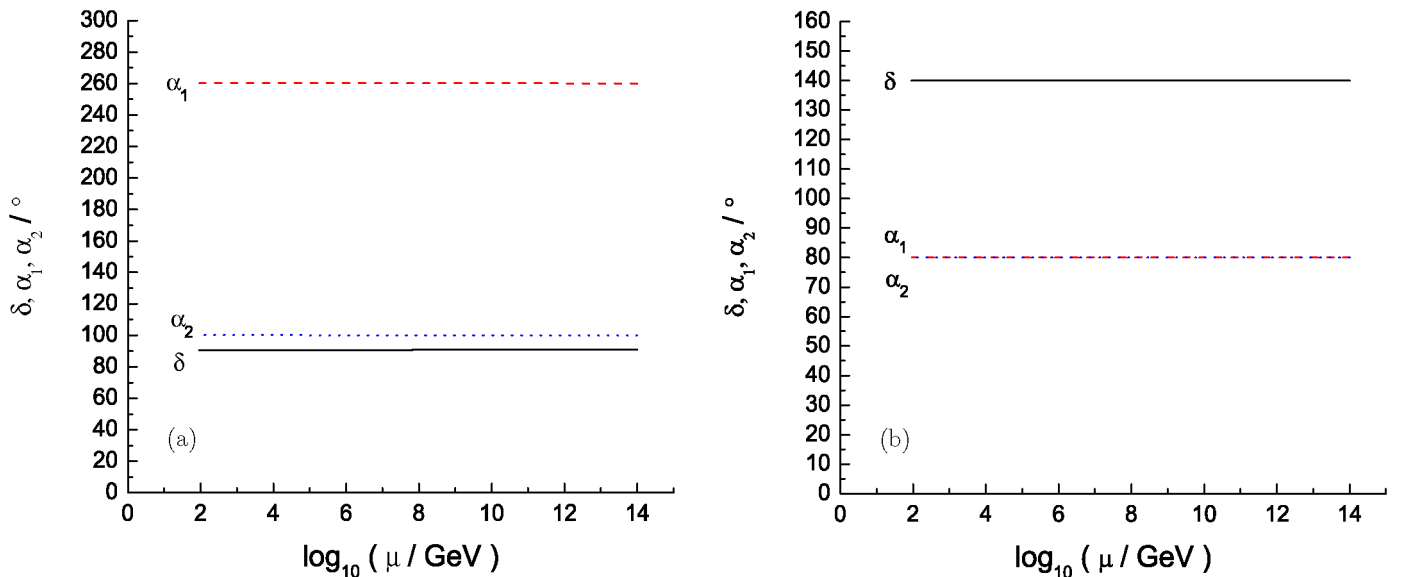


Fig. 1. The RGE running behaviors of three CP-violating phases of V'_0 from Δ_{NP} to Δ_{EW} in the SM. The input and output values of other relevant parameters can be found from Table 1.

⁴ Our numerical calculations follow a “running and diagonalizing” procedure [18]: we first compute the RGE evolution of lepton mass matrices and then extract their mass eigenvalues and flavor mixing parameters at Δ_{EW} . Because $\theta_{13} = 0$ holds exactly at Δ_{NP} and δ is always associated with s_{13} in the chosen parametrization of V , any initial input of δ is allowed but it does not take any effect in the RGE running. The finite running result of δ is actually attributed to the initial values of two Majorana phases α_1 and α_2 .

RGE correction to the CP-violating phase δ arises from the term proportional to $1/s_{13}$ on the right-hand side of Eq. (8). It turns out that δ keeps on staying at its *quasi-fixed point* (see Fig. 1 for illustration). In contrast, the running of α_1 and α_2 is so tiny that they are essentially stable between Λ_{NP} and Λ_{EW} .

We find that the analytical approximation made in Eq. (8) together with Eq. (9) is helpful to understand the quasi-fixed point of the CP-violating phase δ in its RGE evolution from Λ_{NP} to Λ_{EW} . At such a quasi-fixed point, the condition $d\delta/dt \approx 0$ should be satisfied. This observation essentially implies that either

$$\sin(\hat{\delta} + \hat{\alpha}_2) - \sin(\hat{\delta} + \hat{\alpha}_1) \approx 0 \quad (10)$$

with $\hat{\alpha}_1 \neq \hat{\alpha}_2$ or

$$\sin(\hat{\delta} + \hat{\alpha}_1) + \sin \hat{\delta} \approx 0 \quad (11)$$

with $\hat{\alpha}_1 \approx \hat{\alpha}_2$ should hold in the leading-order approximation; i.e., the $1/s_{13}$ term in the expression of $d\delta/dt$ must be strongly suppressed around the quasi-fixed point, where the values of three CP-violating phases are denoted by $\hat{\delta}$, $\hat{\alpha}_1$ and $\hat{\alpha}_2$. The simple solutions to Eqs. (10) and (11) are

$$\hat{\delta} \approx -\frac{1}{2}(\hat{\alpha}_1 + \hat{\alpha}_2) + \left(n + \frac{1}{2}\right)\pi \quad (\hat{\alpha}_1 \neq \hat{\alpha}_2) \quad (12)$$

and

$$\hat{\delta} \approx -\frac{\hat{\alpha}_1}{2} + n\pi \quad (\hat{\alpha}_1 \approx \hat{\alpha}_2) \quad (13)$$

(for $n = 0, \pm 1, \pm 2, \dots$), respectively. These two possibilities correspond to the numerical examples given in cases II and III in Table 1 or Fig. 1(a) and (b).

The tiny magnitude of θ_{13} at Λ_{EW} implies that it is easy to rule out V_0 or V'_0 at Λ_{NP} , after $\theta_{13} \neq 0^\circ$ is experimentally established. Indeed, the sensitivity of a few currently-proposed reactor neutrino experiments to θ_{13} is at the level of $\theta_{13} \sim 3^\circ$ or $\sin^2 2\theta_{13} \sim 0.01$ [23]. Since θ_{13} is considerably suppressed, as shown in our numerical examples, it will be extremely difficult to measure δ (even if $\delta \sim \pm 90^\circ$) in any long-baseline neutrino oscillation experiments. In this case, only the neutrinoless double-beta decay could be used to distinguish V'_0 from V_0 , because its effective mass $\langle m \rangle_{ee}$ is sensitive to the Majorana phases.⁵ We know that V_0 predicts $\langle m \rangle_{ee} \approx m_1$ and V'_0 yields

$$\langle m \rangle'_{ee} \approx m_1 \sqrt{1 - \sin^2 2\theta_{12} \sin^2 \frac{\alpha_2 - \alpha_1}{2}} \quad (14)$$

in the assumption that three neutrino masses are nearly degenerate. The ratio of $\langle m \rangle'_{ee}$ to $\langle m \rangle_{ee}$ may take its minimal value $[\langle m \rangle'_{ee}/\langle m \rangle_{ee}]_{\min} \approx \cos 2\theta_{12} \approx 0.34$ (for $\theta_{12} \approx 35^\circ$), when $\alpha_2 - \alpha_1 \approx \pm\pi$ is satisfied.

We remark that the RGE running behaviors of V_0 and V'_0 are quite different in the SM. This difference can definitely affect the model building in understanding the origin of V_0 or V'_0 . It is worth mentioning that a particular mass model of charged leptons and neutrinos with the non-Abelian symmetry A_4 , from which V'_0 can be derived at a superhigh energy scale, has been proposed and discussed in Ref. [11]. Its low-energy consequences are actually within the scope of our generic RGE analysis.

(B) *In the MSSM.* We have pointed out that the mixing angle θ_{12} is quite sensitive to radiative corrections, and it always runs to a bigger value at Λ_{EW} from the initial value $\theta_{12} \approx 35.3^\circ$ at Λ_{NP} in the MSSM. Considering the tri-bimaximal neutrino mixing pattern V_0 and taking $\tan \beta = 10$ as a typical input, we find that the RGE running result of θ_{12} at Λ_{EW} exceeds its experimental upper bound (i.e., $\theta_{12} \leq 38^\circ$ at the 99% confidence level [6]). We can therefore conclude that a neutrino mass model predicting V_0 at Λ_{NP} is in general disfavored in the MSSM.

This situation will change, if the generalized tri-bimaximal neutrino mixing pattern V'_0 is concerned. The reason is simply that the increase of θ_{12} during its RGE running can be controlled by the Majorana phase factor $\cos^2(\alpha_2 - \alpha_1)$, as shown in Eq. (7). Hence the proper fine-tuning of $(\alpha_2 - \alpha_1)$ will allow θ_{12} to mildly evolve into its experimental range $30^\circ \leq \theta_{12} \leq 38^\circ$ at Λ_{EW} . Given $\tan \beta = 10$, an approximate bound on $(\alpha_2 - \alpha_1)$ is found to be $154^\circ \leq |\alpha_2 - \alpha_1| \leq 206^\circ$ in our calculation. We present an explicit numerical example in Table 2 and Fig. 2, just for the purpose of illustration.

As a direct consequence of $\theta_{13} = 0^\circ$ at Λ_{NP} , a very significant RGE correction to the CP-violating phase δ arises from the term proportional to $1/s_{13}$ in $d\delta/dt$. Thus δ can keep on staying at its *quasi-fixed point* in the MSSM, just like the case in the SM. Following the discussions given above, one may approximately arrive at the relations given in Eqs. (12) and (13) at the quasi-fixed point. The latter possibility has been ruled out by taking into account the evolution of θ_{12} , and the former possibility is illustrated in Table 2 or Fig. 2, where the rather mild running behaviors of α_1 and α_2 can also be seen. In this example, $\theta_{13} \approx 1.4^\circ$

⁵ We have $\langle m \rangle_{ee} = |m_1 c_{12}^2 c_{13}^2 e^{i\alpha_1} + m_2 s_{12}^2 c_{13}^2 e^{i\alpha_2} + m_3 s_{13}^2 e^{-2i\delta}|$ by using the parametrization of V given in Eq. (1). This result implies that it is actually ill to refer to δ as the Dirac phase in this “standard” parametrization. A different phase convention of V has been proposed in Ref. [24] to forbid δ to appear in $\langle m \rangle_{ee}$.

Table 2
Radiative corrections to V'_0 from $\Lambda_{\text{NP}} \sim 10^{14}$ GeV to $\Lambda_{\text{EW}} \sim 10^2$ GeV in the MSSM. In our numerical calculation, $\tan \beta = 10$ has typically been input. Note that δ is not well defined in the $\theta_{13} = 0$ limit at Λ_{NP} , but its running behavior is independent of this ambiguity and is fixed by the initial values of α_1 and α_2

Parameter	Input at Λ_{NP}	Output at Λ_{EW}
m_1 (eV)	0.241	0.201
Δm_{21}^2 (10^{-5} eV ²)	17.0	8.19
Δm_{31}^2 (10^{-3} eV ²)	3.3	2.21
θ_{12}	35.26°	36.38°
θ_{23}	45.0°	46.22°
θ_{13}	0°	1.367°
δ	—	−77.85°
α_1	260.0°	245.17°
α_2	100.0°	92.27°

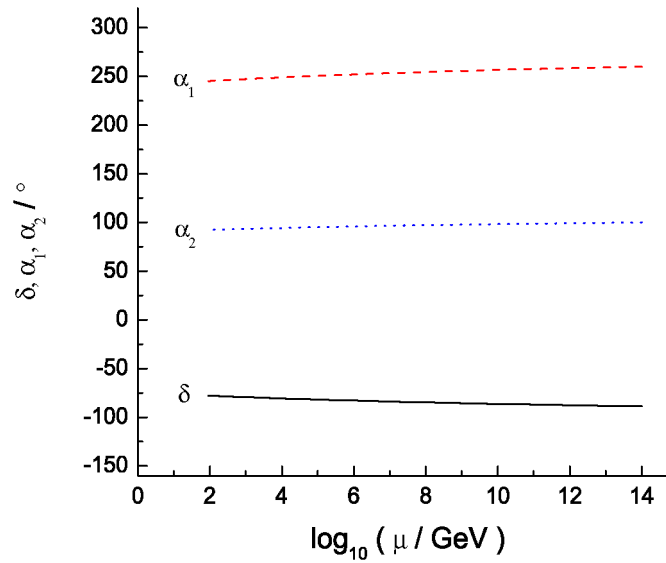


Fig. 2. The RGE running behaviors of three CP-violating phases of V'_0 from Λ_{NP} to Λ_{EW} in the MSSM. The input and output values of other relevant parameters can be found from Table 2.

together with $\delta \approx -78^\circ$ can be radiatively generated at Λ_{EW} . This result implies that the magnitude of the Jarlskog invariant (i.e., $\mathcal{J} = s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^2 \sin \delta$) can be as large as about 0.6%, probably leading to the observable CP-violating effect in the future long-baseline neutrino oscillation experiments.

Let us stress that the quasi-fixed point in the RGE evolution of δ is in general unavoidable for those neutrino mixing patterns with $\theta_{13} = 0$. The above analytic understanding of such a quasi-fixed point is quite instructive and helpful for specific model building. If the phase convention of V in Eq. (1) is replaced by that proposed in [24], its corresponding Majorana phases $\rho = \delta + \alpha_1/2$ and $\sigma = \delta + \alpha_2/2$ will also have the quasi-fixed points in their RGE evolution. This point can easily be understood with the help of Eq. (8): the dominant term of $d\delta/dt$ (proportional to $1/s_{13}$) will enter $d\rho/dt$ and $d\sigma/dt$, such that the RGE running behaviors of δ , ρ and σ are essentially identical [19]. In short, the “standard” parametrization of V taken in Eq. (1) is more convenient in discussing the issue of quasi-fixed points, while the phase convention of V advocated in Ref. [24] is more convenient in discussing the neutrinoless double-beta decay (i.e., $\langle m \rangle_{ee}$ is dependent on ρ and σ but independent of δ).

4. We have argued that the tri-bimaximal neutrino mixing pattern V_0 or its generalized counterpart V'_0 is very likely to result from an underlying flavor symmetry, and this new symmetry is most likely to be realized at a superhigh energy scale. Supposing that this new physics scale is close to the neutrino seesaw scale ($\sim 10^{14}$ GeV), we have calculated the one-loop RGE effects on V_0 and V'_0 in their evolution down to the electroweak scale ($\sim 10^2$ GeV) in the working assumption that three neutrino masses are nearly degenerate. It is found that three mixing angles of V_0 or V'_0 are essentially insensitive to radiative corrections in the SM. In the MSSM, however, V_0 is in general disfavored and V'_0 can be compatible with current neutrino oscillation data if its two Majorana phases α_1 and α_2 are properly fine-tuned. We have also shown that it is possible to radiatively generate the CP-violating phase δ from α_1 and α_2 , and δ may keep on staying at its quasi-fixed point in both the SM and the MSSM.

Although a detailed RGE analysis of the tri-bimaximal neutrino mixing scenario has not been done before, some of our results have actually been observed by some other authors in their analyses of radiative corrections to the neutrino mass spectrum and realistic lepton flavor mixing patterns, whose forms are more or less similar to the tri-bimaximal neutrino mixing pattern V_0 . However, our present work is very non-trivial in two important aspects: (1) we generalize V_0 to V'_0 with two arbitrary Majorana phases, because the latter is more interesting and can naturally appear in some specific neutrino mass models; (2) we explore the quasi-fixed point in the RGE evolution of δ and present an analytic understanding of this non-trivial phenomenon.

The afore-mentioned RGE running behaviors of V_0 and V'_0 are expected to be useful for model building at a superhigh energy scale. A similar study can be extended to some other interesting neutrino mixing patterns. For example, the pattern

$$U_0 = \begin{pmatrix} \sqrt{3}/2 & 1/2 & 0 \\ -\sqrt{2}/4 & \sqrt{6}/4 & \sqrt{2}/2 \\ \sqrt{2}/4 & -\sqrt{6}/4 & \sqrt{2}/2 \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (15)$$

which has $\theta_{12} = 30^\circ$, $\theta_{23} = 45^\circ$ and $\theta_{13} = 0^\circ$ [25], is rather analogous to the generalized tri-bimaximal neutrino mixing pattern V'_0 . If U_0 is derived from an underlying flavor symmetry at an energy scale close to the seesaw scale, then its sensitivity to radiative corrections must be very similar to that of V'_0 .

To examine whether such a special lepton mixing scenario is viable or not in a high-energy neutrino mass model, it is crucial to measure the smallest mixing angle θ_{13} and the CP-violating phase δ in the future neutrino oscillation experiments. Furthermore, any experimental information about the Majorana phases α_1 and α_2 is welcome and extremely important, in order to distinguish one model from another through their different sensitivities to radiative corrections.

Acknowledgements

We would like to thank J.W. Mei for his very nice *Mathematica* program and H. Zhang for very useful physical discussions and technical helps. This work was supported in part by the National Natural Science Foundation of China.

References

- [1] SNO Collaboration, Q.R. Ahmad, et al., Phys. Rev. Lett. 89 (2002) 011301.
- [2] For a review, see: C.K. Jung, et al., Annu. Rev. Nucl. Part. Sci. 51 (2001) 451.
- [3] KamLAND Collaboration, K. Eguchi, et al., Phys. Rev. Lett. 90 (2003) 021802;
CHOOZ Collaboration, M. Apollonio, et al., Phys. Lett. B 420 (1998) 397;
Palo Verde Collaboration, F. Boehm, et al., Phys. Rev. Lett. 84 (2000) 3764.
- [4] K2K Collaboration, M.H. Ahn, et al., Phys. Rev. Lett. 90 (2003) 041801.
- [5] Particle Data Group, S. Eidelman, et al., Phys. Lett. B 592 (2004) 1.
- [6] A. Strumia, F. Vissani, Nucl. Phys. B 726 (2005) 294.
- [7] For recent reviews with extensive references, see: H. Fritzsch, Z.Z. Xing, Prog. Part. Nucl. Phys. 45 (2000) 1;
S.F. King, Rep. Prog. Phys. 67 (2004) 107;
G. Altarelli, F. Feruglio, New J. Phys. 6 (2004) 106.
- [8] P.F. Harrison, D.H. Perkins, W.G. Scott, Phys. Lett. B 530 (2002) 167;
Z.Z. Xing, Phys. Lett. B 533 (2002) 85;
P.F. Harrison, W.G. Scott, Phys. Lett. B 535 (2002) 163;
X.G. He, A. Zee, Phys. Lett. B 560 (2003) 87;
E. Ma, Phys. Rev. Lett. 90 (2003) 221802;
C.I. Low, R.R. Volkas, Phys. Rev. D 68 (2003) 033007;
N. Li, B.Q. Ma, Phys. Rev. D 71 (2005) 017302;
G. Altarelli, F. Feruglio, Nucl. Phys. B 720 (2005) 64;
F. Plentinger, W. Rodejohann, hep-ph/0507143.
- [9] E. Ma, Phys. Rev. D 70 (2004) 031901;
E. Ma, New J. Phys. 6 (2004) 104, hep-ph/0508099.
- [10] G. Altarelli, F. Feruglio, in Ref. [8], and references therein.
- [11] K.S. Babu, X.G. He, hep-ph/0507217.
- [12] A. Zee, hep-ph/0508278.
- [13] See, e.g., W. Grimus, A.S. Joshipura, S. Kaneko, L. Lavoura, M. Tanimoto, JHEP 0407 (2004) 078;
W. Grimus, L. Lavoura, JHEP 0508 (2005) 013;
W. Grimus, L. Lavoura, J. Phys. G 31 (2005) 693;
W. Grimus, L. Lavoura, Nucl. Phys. B 713 (2005) 151;
M. Lindner, M. Ratz, M.A. Schmidt, hep-ph/0506280.
- [14] P.H. Chankowski, Z. Pluciennik, Phys. Lett. B 316 (1993) 312;
K.S. Babu, C.N. Leung, J. Pantaleone, Phys. Lett. B 319 (1993) 191;
M. Tanimoto, Phys. Lett. B 360 (1995) 41;
S. Antusch, M. Drees, J. Kersten, M. Lindner, M. Ratz, Phys. Lett. B 519 (2001) 238, only this item yields the correct beta-function for the SM.
- [15] P. Minkowski, Phys. Lett. B 67 (1977) 421;

- T. Yanagida, in: O. Sawada, A. Sugamoto (Eds.), *Proceedings of the Workshop on Unified Theory and the Baryon Number of the Universe*, KEK, Tsukuba, 1979, p. 95;
- M. Gell-Mann, P. Ramond, R. Slansky, in: F. van Nieuwenhuizen, D. Freedman (Eds.), *Supergravity*, North-Holland, Amsterdam, 1979, p. 315;
- S.L. Glashow, in: M. Lévy, et al. (Eds.), *Quarks and Leptons*, Plenum, New York, 1980, p. 707;
- R.N. Mohapatra, G. Senjanovic, *Phys. Rev. Lett.* 44 (1980) 912.
- [16] For recent comprehensive works, see: J.A. Casas, J.R. Espinosa, A. Ibarra, I. Navarro, *Nucl. Phys. B* 573 (2000) 652;
- P.H. Chankowski, S. Pokorski, *Int. J. Mod. Phys. A* 17 (2002) 575;
- S. Antusch, J. Kersten, M. Lindner, M. Ratz, M.A. Schmidt, *JHEP* 0503 (2005) 024;
- J.W. Mei, *Phys. Rev. D* 71 (2005) 073012.
- [17] See, e.g., J.A. Casas, J.R. Espinosa, A. Ibarra, I. Navarro, *Nucl. Phys. B* 556 (1999) 3;
- J.A. Casas, J.R. Espinosa, A. Ibarra, I. Navarro, *Nucl. Phys. B* 569 (2000) 82;
- S. Antusch, M. Drees, J. Kersten, M. Lindner, M. Ratz, *Phys. Lett. B* 525 (2002) 130;
- S. Antusch, J. Kersten, M. Lindner, M. Ratz, *Phys. Lett. B* 538 (2002) 87;
- J.W. Mei, Z.Z. Xing, *Phys. Rev. D* 70 (2004) 053002;
- S. Antusch, J. Kersten, M. Lindner, M. Ratz, M.A. Schmidt, in Ref. [16];
- J.W. Mei, in Ref. [16];
- J. Ellis, A. Hektor, M. Kadastik, K. Kannike, M. Raidal, hep-ph/0506122.
- [18] S. Antusch, J. Kersten, M. Lindner, M. Ratz, *Nucl. Phys. B* 674 (2003) 401.
- [19] S. Luo, J. Mei, Z.Z. Xing, *Phys. Rev. D* 72 (2005) 053014.
- [20] H. Fritzsch, Z.Z. Xing, *Phys. Lett. B* 372 (1996) 265;
- H. Fritzsch, Z.Z. Xing, *Phys. Lett. B* 440 (1998) 313;
- H. Fritzsch, Z.Z. Xing, *Phys. Rev. D* 61 (2000) 073016;
- H. Fritzsch, Z.Z. Xing, *Phys. Lett. B* 598 (2004) 237.
- [21] N. Haba, Y. Matsui, N. Okamura, M. Sugiura, *Prog. Theor. Phys.* 103 (2000) 145;
- N. Haba, N. Okamura, M. Sugiura, *Prog. Theor. Phys.* 103 (2000) 367;
- N. Haba, Y. Matsui, N. Okamura, *Eur. Phys. J. C* 17 (2000) 513.
- [22] C. Jarlskog, *Phys. Rev. Lett.* 55 (1985) 1039.
- [23] See, e.g., Y.F. Wang, hep-ex/0411028.
- [24] H. Fritzsch, Z.Z. Xing, *Phys. Lett. B* 517 (2001) 363;
- Z.Z. Xing, *Int. J. Mod. Phys. A* 19 (2004) 1.
- [25] Z.Z. Xing, *J. Phys. G* 29 (2003) 2227;
- C. Giunti, hep-ph/0209103.